Open Quantum Systems

Lecture III:

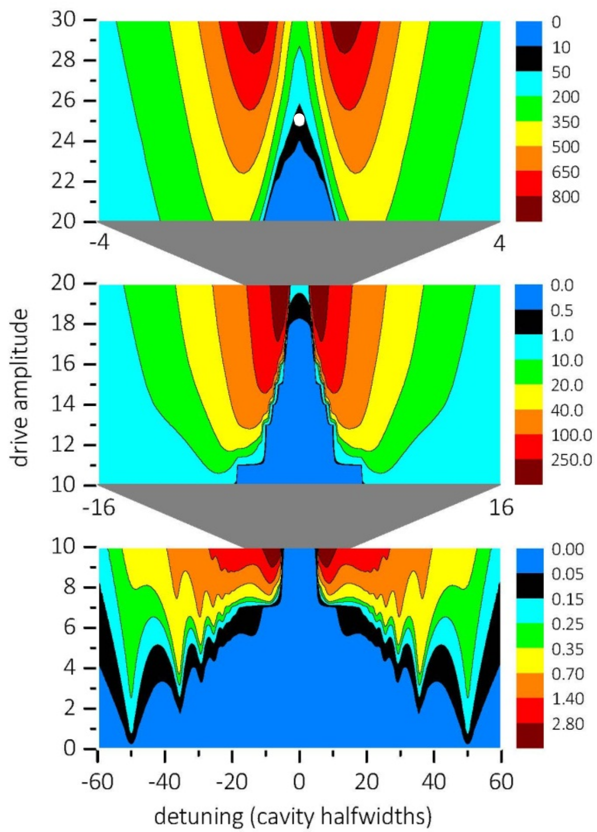
Master Equations, Inputs & Outputs, Quantum Trajectories – Part A

H. J. Carmichael

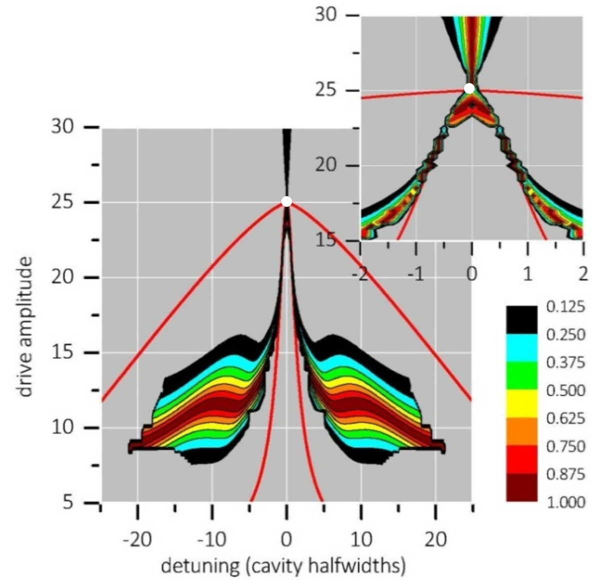
University of Auckland

dipole coupling $= 50 \times$ cavity linewidth

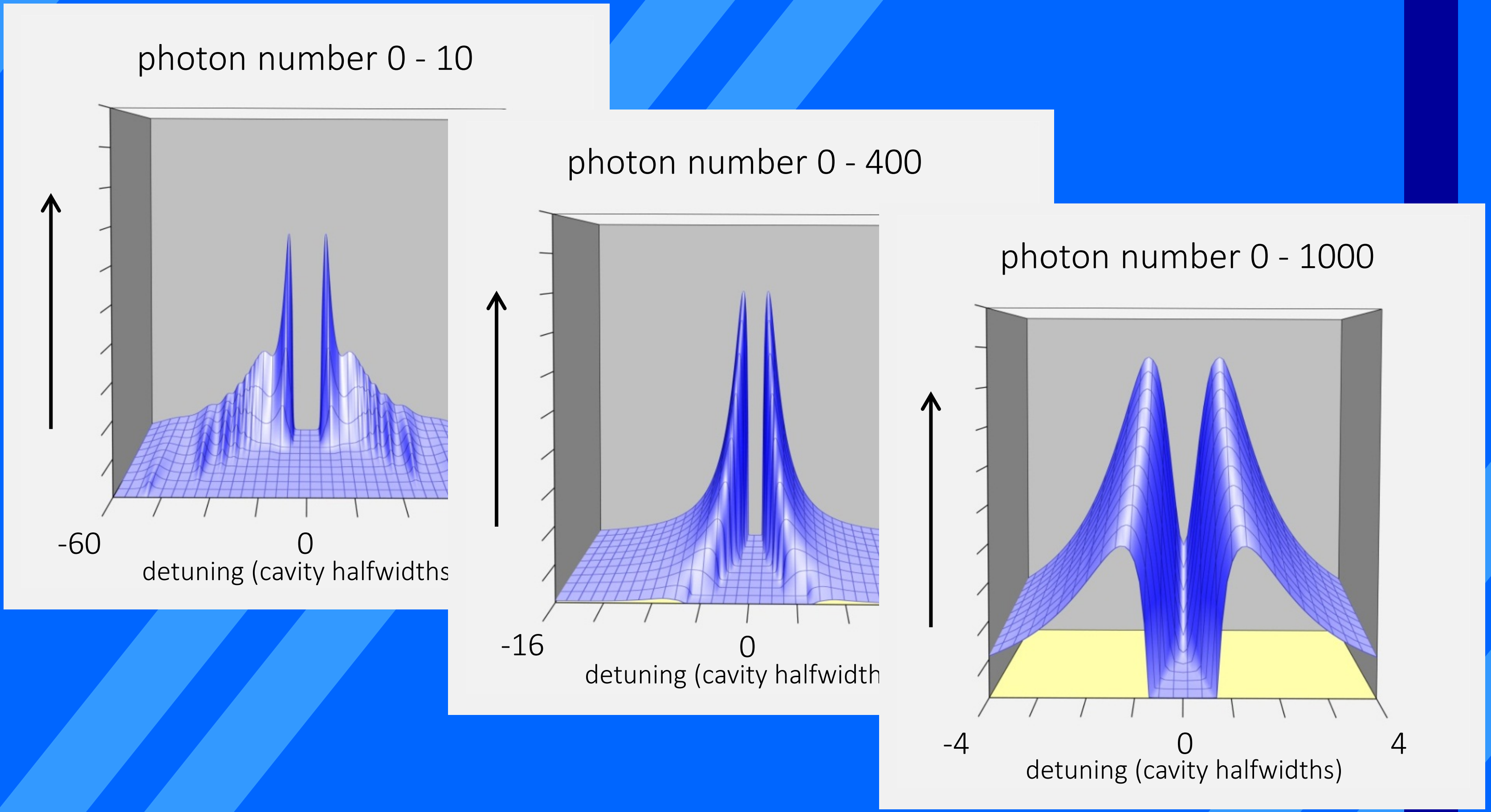
photon number



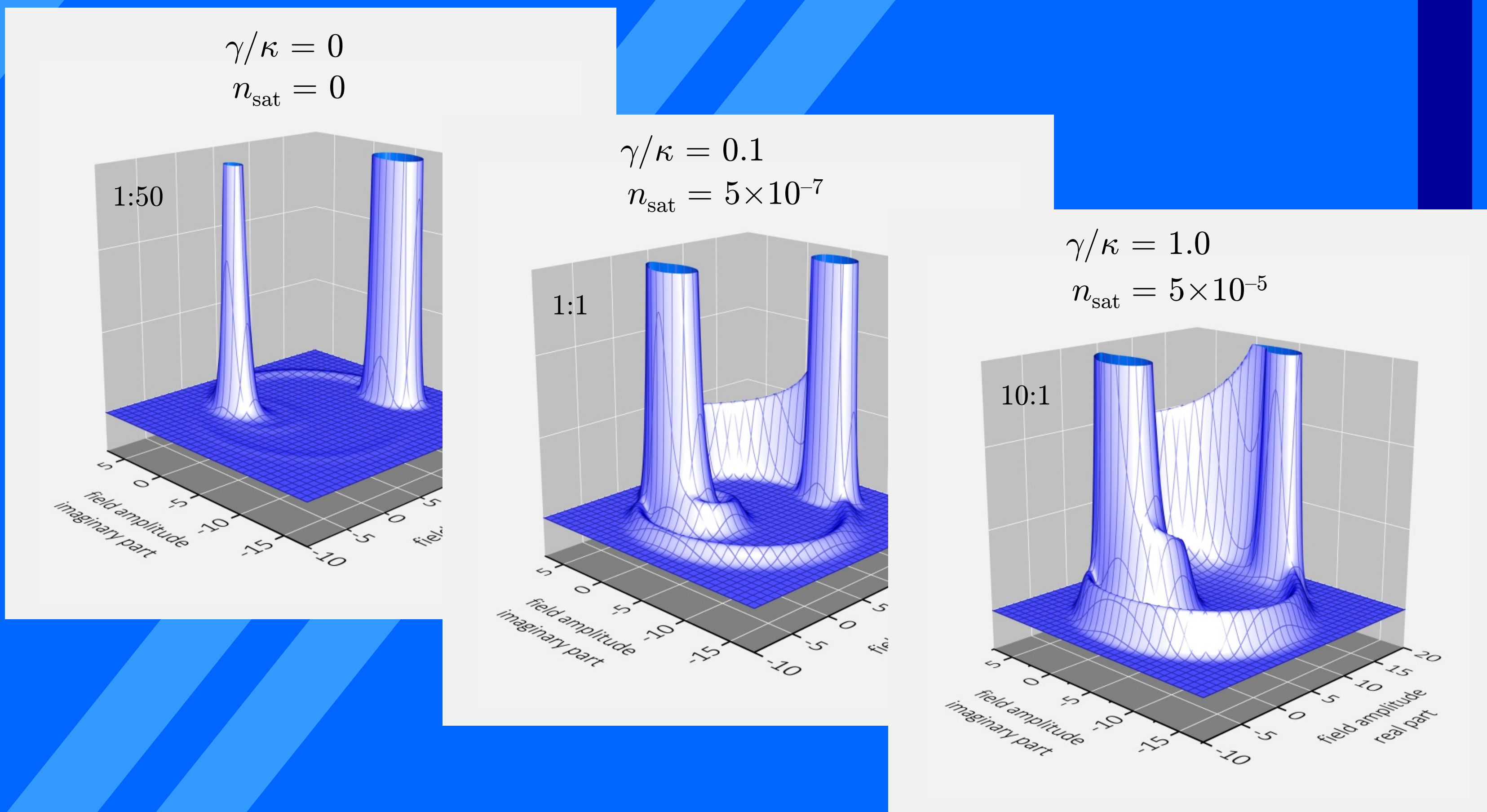
bimodality



BREAKDOWN OF BLOCKADE



LADDER SWITCHING



MASTER EQUATION – OPEN JAYNES-CUMMINGS MODEL

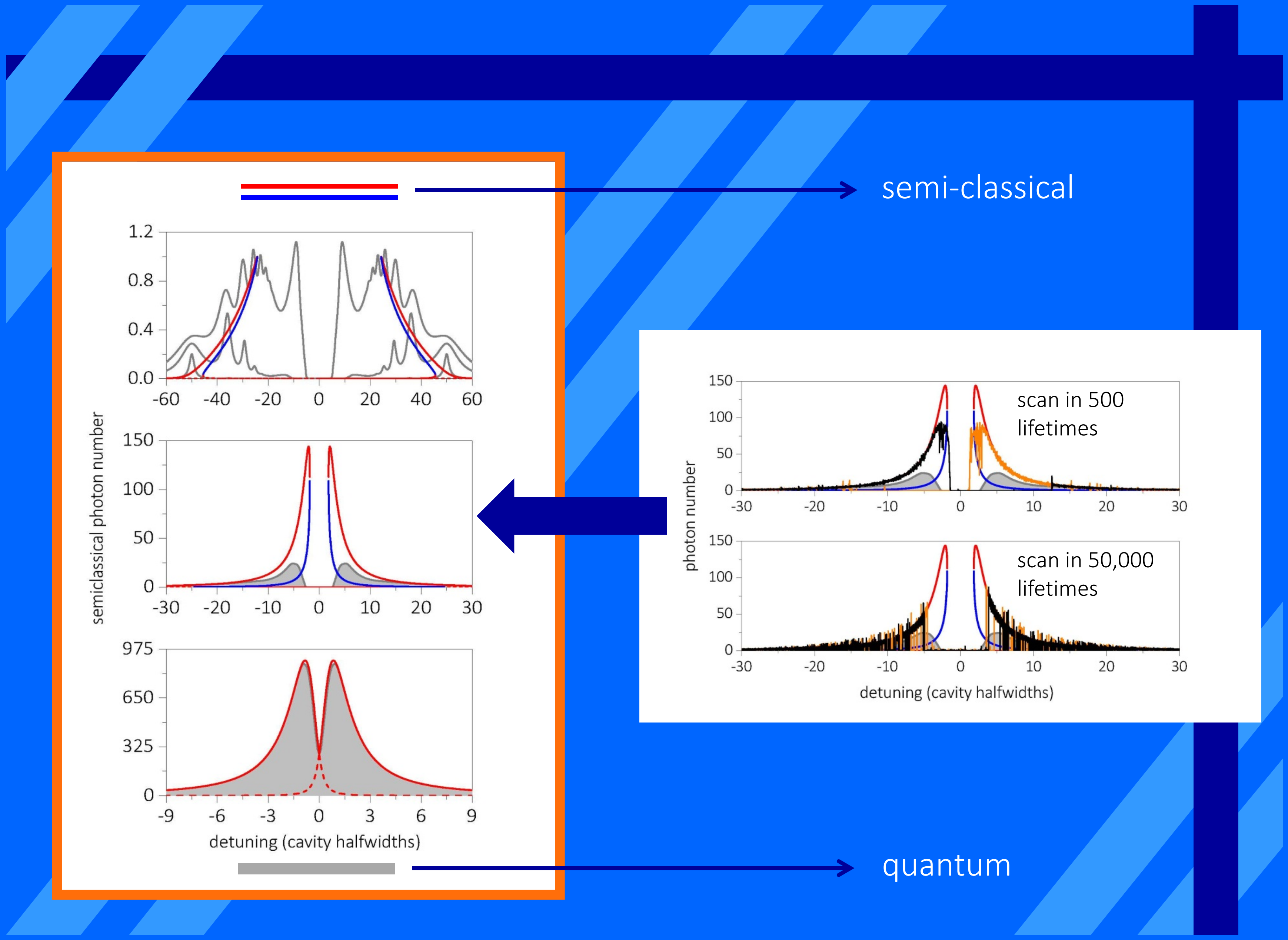
input:
coherent drive

output:
cavity loss

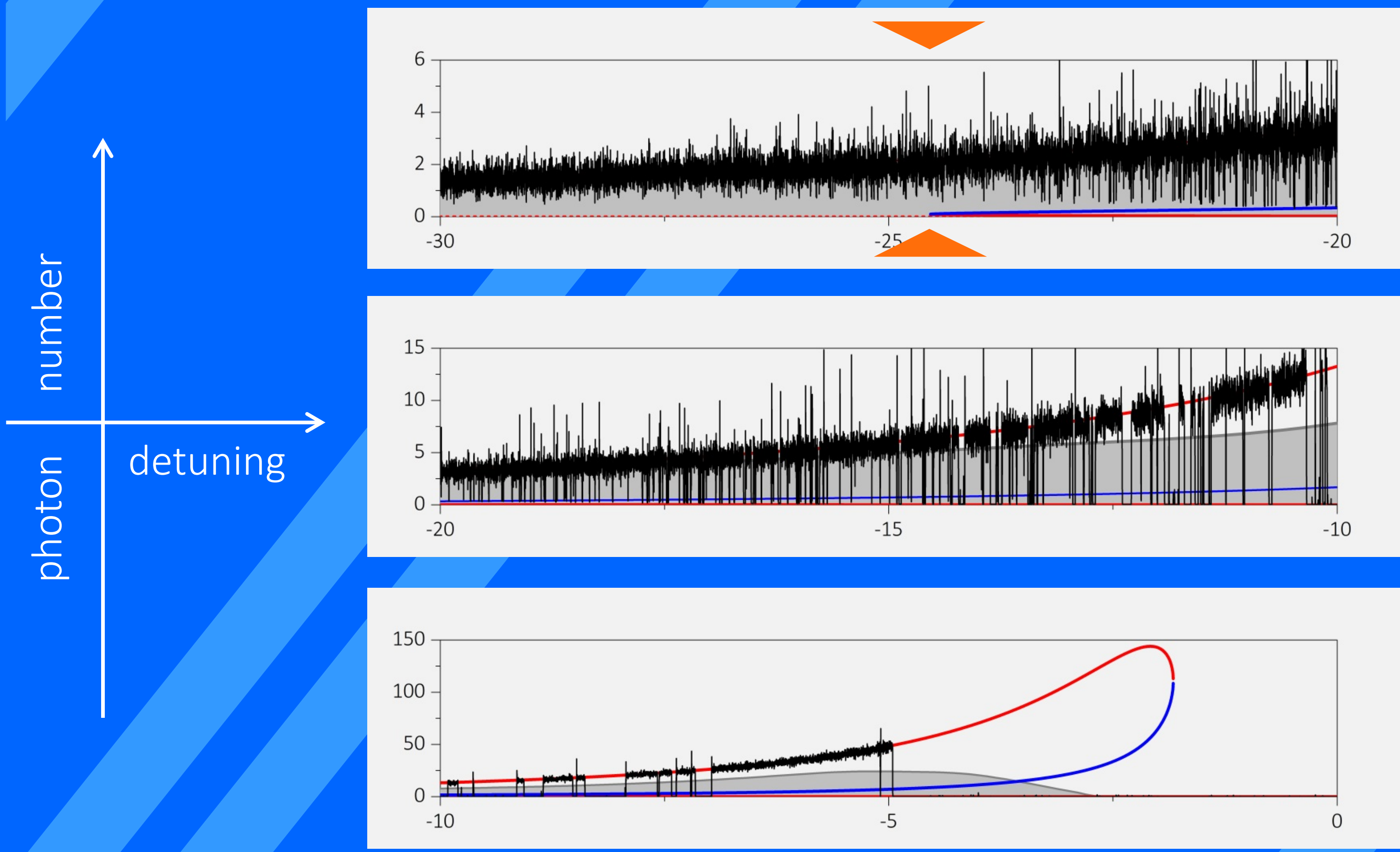
$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H_{JC}^{\text{driven}}(t), \rho] + \kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)$$

output:
spontaneous emission

$$+\frac{\gamma}{2}(2|\mathrm{g}\rangle\langle\mathrm{e}|\rho|\mathrm{e}\rangle\langle\mathrm{g}| - |\mathrm{e}\rangle\langle\mathrm{e}|\rho - \rho|\mathrm{e}\rangle\langle\mathrm{e}|)$$


semi-classical
quantum
semiclassical photon number
detuning (cavity halfwidths)
photon number
scan in 500 lifetimes
scan in 50,000 lifetimes
detuning (cavity halfwidths)

QUANTUM TRAJECTORY SIMULATION



Master equations, Langevin equations, inputs & outputs

Quantum trajectories – unraveling the master equation

Quantum trajectories – Monte-Carlo simulation

Master equations, Langevin equations, inputs & outputs

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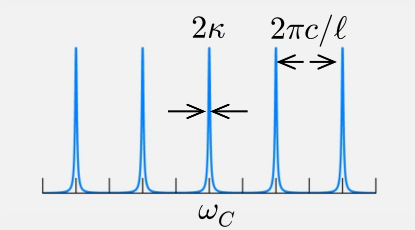
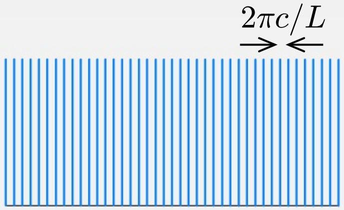
MODEL

SYSTEM

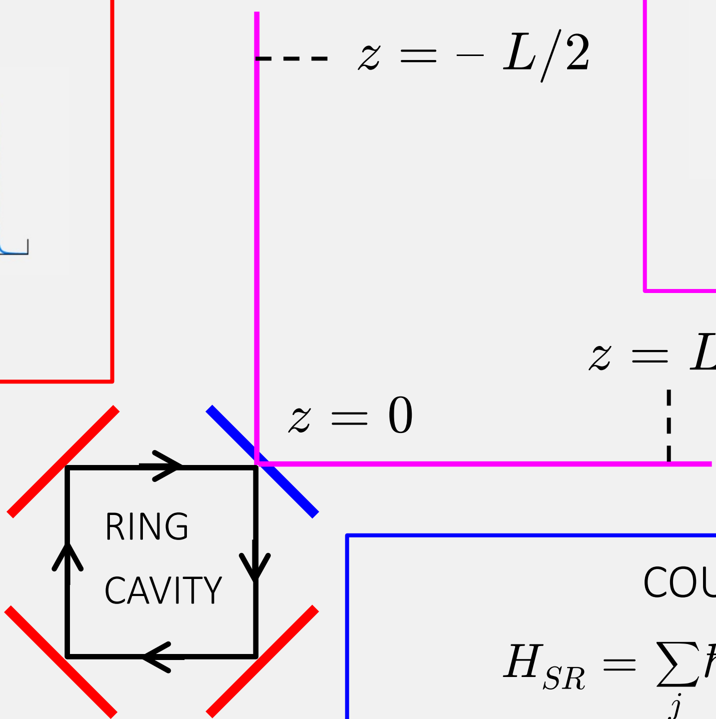


$$H_S = \hbar\omega_C a^\dagger a$$

RESERVOIR



$$H_R = \sum_j \hbar\omega_j r_j^\dagger r_j$$



COUPLING

$$H_{SR} = \sum_j \hbar\kappa_j r_j a^\dagger + \text{h.c.}$$

SCHRÖDINGER PICTURE – LINDBLAD MASTER EQUATION

$$\frac{d\widetilde{\chi}}{dt} = \frac{1}{i\hbar}[\widetilde{H}_{SR}(t),\chi(0)] - \frac{1}{\hbar^2}\int_0^t dt'[\widetilde{H}_{SR}(t),[\widetilde{H}_{SR}(t'),\widetilde{\chi}(t')]]$$

$$\widetilde{\chi}(t') \xrightarrow{\text{Born}} R_0\widetilde{\rho}(t') \xrightarrow{\text{Markov}} R_0\widetilde{\rho}(t)$$

$$\frac{d\widetilde{\rho}}{dt} = -\frac{1}{\hbar^2}\int_0^t dt'\,\mathrm{tr}_R[\widetilde{H}_{SR}(t),[\widetilde{H}_{SR}(t'),\widetilde{\rho}(t)]]$$

Fermi golden rule:

decay rate 2κ

$$\sum_j |\kappa_j|^2 e^{i(\omega_j - \omega_C)(t-t')} \to \underline{2\pi g(\omega_C)|\kappa_{\omega_j = \omega_C}|^2}\, \delta(t-t')$$

Reservoir correlation functions:

$$\sum_{j,k} \kappa_j^* \kappa_k \langle r_k r_j^\dagger \rangle_{R_0} e^{i\omega_j t} e^{-i\omega_k t'} \to \underline{2\kappa(\bar{n}+1)\delta(t-t')}$$

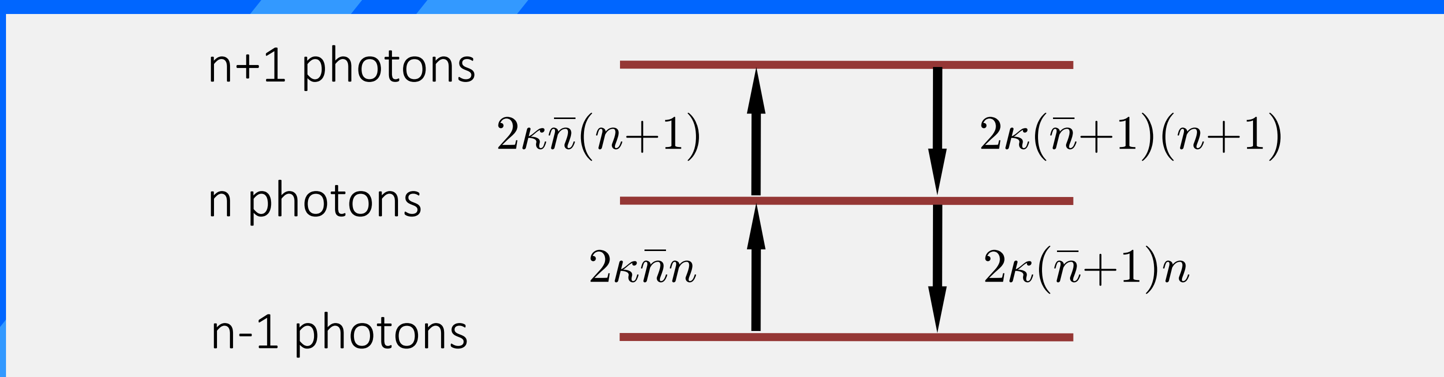
$$\sum_{j,k} \kappa_j^* \kappa_k \langle r_j^\dagger r_k \rangle_{R_0} e^{i\omega_j t} e^{-i\omega_k t'} \to \underline{2\kappa\bar{n}\delta(t-t')}$$

down transitions

$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H_S,\rho] + \kappa(\bar{n}+1)(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)$$

up transitions

$$+\ \kappa\bar{n}(2a^\dagger\rho a - aa^\dagger\rho - \rho aa^\dagger)$$



HEISENBERG PICTURE – QUANTUM LANGEVIN EQUATION

$$\frac{dr_j}{dt} = \underline{-i\omega_j r_j - i\kappa_j^* a}$$

$$\frac{da}{dt} = \underline{\frac{1}{i\hbar}[a, H_S] - i\sum_j \kappa_j r_j}$$

Formally integrate:

$$\widetilde{r}_j(t) = r_j(0) - ie^{i(\omega_j - \omega_C)t}\kappa_j^* \int_0^t dt' \, \widetilde{a}(t') e^{-i(\omega_j - \omega_C)(t - t')}$$

Substitute – use Fermi golden rule:

output:
cavity loss

input

$$\frac{da}{dt} = \underline{\frac{1}{i\hbar}[a, H_S] - \kappa a - i\sqrt{2\kappa}\sqrt{\frac{c}{L}}\sum_j r_j e^{-i\omega_j t}}$$

INPUT & OUTPUT FIELDS

$$E^{(+)}(z,t) = i \sum_j \sqrt{\frac{\hbar\omega_j}{2\epsilon_0 AL}}\, r_j(t) e^{i(\omega_j/c)z}$$

Substitute for $r_j(t)$

Multiply by $\sqrt{2\epsilon_0 AL/\hbar\omega_C}$ – photon flux units

Fermi golden rule

$$E^{(+)}(z,t) \to \begin{cases} \mathcal{E}_{\text{in}}^{(+)}(z,t) \\ \mathcal{E}_{\text{out}}^{(+)}(z,t) = \mathcal{E}_{\text{in}}^{(+)}(z,t) + \sqrt{2\kappa}\, a(t-z/c) \end{cases}$$

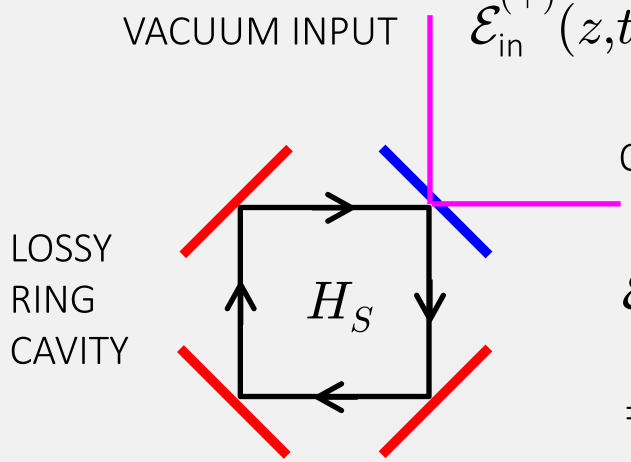
Master equations, Langevin equations, inputs & outputs

Quantum trajectories – unraveling the master equation

Quantum trajectories – Monte-Carlo simulation

MASTER EQUATION/INPUTS & OUTPUTS

$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H_S, \rho] + \underline{\kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)}$$



$\mathcal{E}_{\text{in}}^{(+)}(z,t)$

$$\mathcal{E}_{\text{out}}^{(+)}(z,t) = \mathcal{E}_{\text{in}}^{(+)}(z,t) + \sqrt{2\kappa}\, a(t-z/c)$$

QUANTUM TRAJECTORY UNRAVELING

$$\frac{d\rho}{dt} = \underline{\frac{1}{i\hbar}[H_S, \rho] + \kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)}$$

$$\frac{d\rho}{dt} = \underline{\frac{1}{i\hbar}(H_S - i\hbar\kappa a^\dagger a)\rho - \frac{1}{i\hbar}\rho(H_S + i\hbar\kappa a^\dagger a)} + \underline{2\kappa a\rho a^\dagger}$$

non-Hermitian H

jump operator J

$$\underline{\frac{d|\widetilde{\psi}\rangle}{dt} = \frac{1}{i\hbar}(H_S - i\hbar\kappa a^\dagger a)|\widetilde{\psi}\rangle}$$

PLUS JUMPS

$$\underline{|\widetilde{\psi}\rangle \to \sqrt{2\kappa}a|\widetilde{\psi}\rangle}$$

DYSON EXPANSION

$$\frac{d\rho}{dt} = \underline{\left[(\mathcal{L} - \mathcal{S}) + \mathcal{S}\right]\rho}$$

$$\rho(t) = \sum_{n=0}^{\infty} \int_0^t dt_n \int_0^{t_n} dt_{n\text{-}1} \cdots \int_0^{t_2} dt_1 \, \mathrm{exp}[(\mathcal{L} - \mathcal{S})(t - t_n)]\,\mathcal{S}$$

$$\times \; \mathrm{exp}[(\mathcal{L} - \mathcal{S})(t_n - t_{n\text{-}1})]\,\mathcal{S} \cdots \mathcal{S}\, \mathrm{exp}[(\mathcal{L} - \mathcal{S})t_1]\rho(0)$$

$$\underline{\frac{1}{i\hbar}\left(H\cdot - \cdot H^{\dagger}\right)} \qquad\qquad \underline{J \cdot J^{\dagger}}$$

A SUGGESTIVE INTERPRETATION

$$|\widetilde{\psi}_{\mathsf{REC}}(t)\rangle = \underline{\exp\left[-\frac{1}{i\hbar}H(t-t_n)\right]J\cdots J\exp\left[-\frac{1}{i\hbar}Ht_1\right]|\psi(0)\rangle}$$

Unraveling as sum over RECORDS:

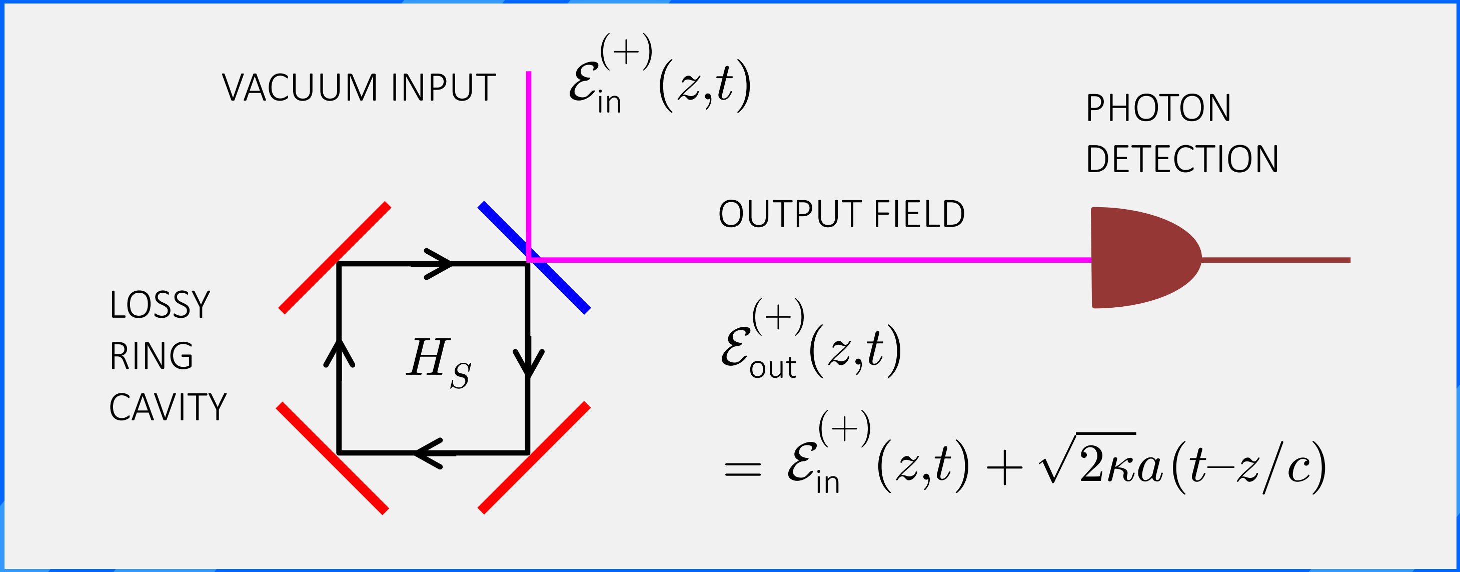
$$\rho(t) = \sum_{n=0}^{\infty}\int_0^t dt_n \int_0^{t_n} dt_{n\text{-}1}\cdots\int_0^{t_2} dt_1\, \underline{P_{\mathsf{REC}}(t)}\,\frac{|\widetilde{\psi}_{\mathsf{REC}}(t)\rangle\langle\widetilde{\psi}_{\mathsf{REC}}(t)|}{\langle\widetilde{\psi}_{\mathsf{REC}}(t)|\widetilde{\psi}_{\mathsf{REC}}(t)\rangle}$$

$$\underline{P_{\mathsf{REC}}(t) = \langle\widetilde{\psi}_{\mathsf{REC}}(t)|\widetilde{\psi}_{\mathsf{REC}}(t)\rangle}$$

RECORD
probability density

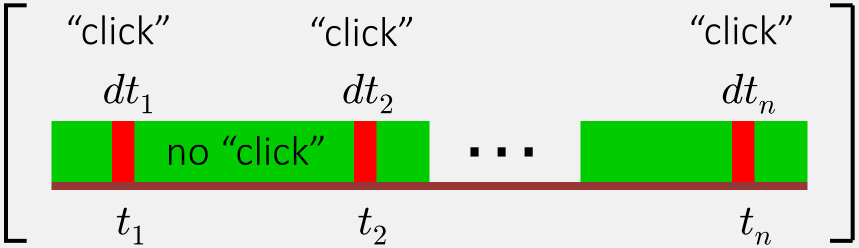
ADD PHOTON DETECTION OF OUTPUTS

$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H_S, \rho] + \underline{\kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)}$$

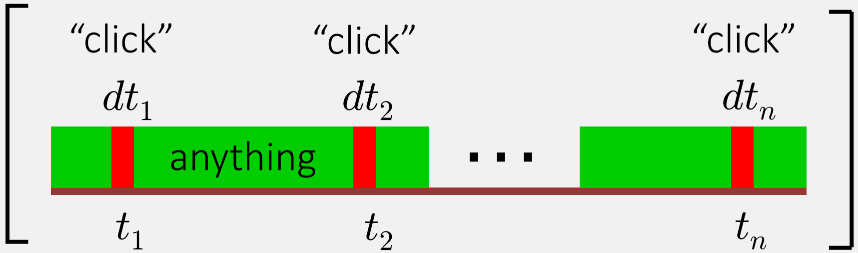


EXCLUSIVE & NONEXCLUSIVE COUNTING PROBABILITIES

exclusive probability = Prob



nonexclusive probability = Prob



MANDEL/GLAUBER/KELLEY-KLEINER

Glauber correlation functions:

$$\left\langle \mathcal{E}_{\text{out}}^{(-)}(t_1) \cdots \mathcal{E}_{\text{out}}^{(-)}(t_n)\, \mathcal{E}_{\text{out}}^{(+)}(t_n) \cdots \mathcal{E}_{\text{out}}^{(+)}(t_1) \right\rangle dt_1 \cdots dt_n$$

nonexclusive

exclusive

$$\left\langle : \exp\left[-\int_0^t dt' \mathcal{E}_{\text{out}}^{(-)}(t')\, \mathcal{E}_{\text{out}}^{(+)}(t') \right] \mathcal{E}_{\text{out}}^{(-)}(t_n)\, \mathcal{E}_{\text{out}}^{(+)}(t_n) \cdots \right.$$

$$\left. \cdots \mathcal{E}_{\text{out}}^{(-)}(t_1)\, \mathcal{E}_{\text{out}}^{(+)}(t_1) \exp\left[-\int_0^{t_1} dt' \mathcal{E}_{\text{out}}^{(-)}(t')\, \mathcal{E}_{\text{out}}^{(+)}(t') \right] : \right\rangle dt_1 \cdots dt_n$$

PROBLEM? – NEGATIVE COUNTING PROBABILITIES!

$$\left\langle : \frac{\left[\int_0^t dt' \mathcal{E}^{(-)}_{\text{out}}(t')\,\mathcal{E}^{(+)}_{\text{out}}(t')\right]^n}{n!} \exp\left[-\int_0^t dt' \mathcal{E}^{(-)}_{\text{out}}(t')\,\mathcal{E}^{(+)}_{\text{out}}(t')\right] : \right\rangle$$

SINGLE MODE & ONE-PHOTON STATE

probability to count zero photons

$$\left\langle : \exp\left[-\int_0^t dt' \overset{\text{single mode}}{\lambda a^\dagger a}\right] : \right\rangle \xrightarrow[\text{state}]{\text{one photon}} 1 - \lambda t$$

MAYBE

M. D. Srinivas
& E. B. Davies
OPTICA ACTA **28**
981-996 (1981)

OPTICA ACTA, 1981, VOL. 28, NO. 7, 981–996

Photon counting probabilities in quantum optics

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Guindy Campas, Madras-600 025, India

and E. B. DAVIES

St. John's College, Oxford OX1 3JP, England

(*Received 5 August 1980; revision received 4 November 1980*)

Abstract. We reconsider various approaches to the quantum theory of photodetection from the point of view of the quantum theory of measurement, and show that important differences between them depend upon the manner in which they take into account the modification of the field distribution produced by the presence of the detector. We show that the Mandel photon counting formula may lead to unphysical results, such as negative probabilities, in some situations just because this modification is not incorporated into the model. We also show that the recently developed quantum theory of continuous measure-

We show that the Mandel photon counting formula may lead to unphysical results, such as negative probabilities, in some situations just because this modification [i.e., modification of the field distribution produced by the presence of the detector] is not incorporated into the model.

MAYBE NOT

L. Mandel
OPTICA ACTA **28**
1447-1450 (1981)

OPTICA ACTA, 1981, VOL. 28, NO. 11, 1447–1450

Comment on 'Photon counting probabilities in quantum optics'

L. MANDEL
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Rochester, New York 14627, U.S.A.

(*Received 9 September 1981*)

Abstract. It is pointed out that some statements in a recent paper by Srinivas and Davies [1], concerning the validity of the so-called Mandel photon counting formula and its relation to another formula derived by the authors, are not correct. The former formula applies to an open system and the latter to a closed system, but conditions for the former are much more commonly encountered in practice.

Briefly, the Mandel formula (3) applies to an open system, in which light falls on the photodetector and any unabsorbed photons propagate away. Formula (1) of Srinivas and Davies [1], on the other hand, applies to a closed system, in which field and detector are both contained in some cavity, and any photons not absorbed by the detector at one time are available for detection at later times.

UNIFICATION – MASTER EQUATION UNRAVELING

$$\mathcal{E}_{\text{out}}^{(+)} \cdot \mathcal{E}_{\text{out}}^{(-)} = \underline{\left[\mathcal{E}_{\text{in}}^{(+)} + \sqrt{2\kappa}a\right] \cdot \left[\mathcal{E}_{\text{in}}^{(-)} + \sqrt{2\kappa}a^{\dagger}\right]}$$

Exclusive probability density

$$\left\langle : \text{exp}\left[-\int_0^t dt'\, \mathcal{E}_{\text{out}}^{(-)}(t')\, \mathcal{E}_{\text{out}}^{(+)}(t')\right] \mathcal{E}_{\text{out}}^{(-)}(t_n)\, \mathcal{E}_{\text{out}}^{(+)}(t_n) \cdots \right.$$

$$\left. \cdots\, \mathcal{E}_{\text{out}}^{(-)}(t_1)\, \mathcal{E}_{\text{out}}^{(+)}(t_1)\, \text{exp}\left[-\int_0^{t_1} dt'\, \mathcal{E}_{\text{out}}^{(-)}(t')\, \mathcal{E}_{\text{out}}^{(+)}(t')\right] : \right\rangle dt_1 \cdots dt_n$$

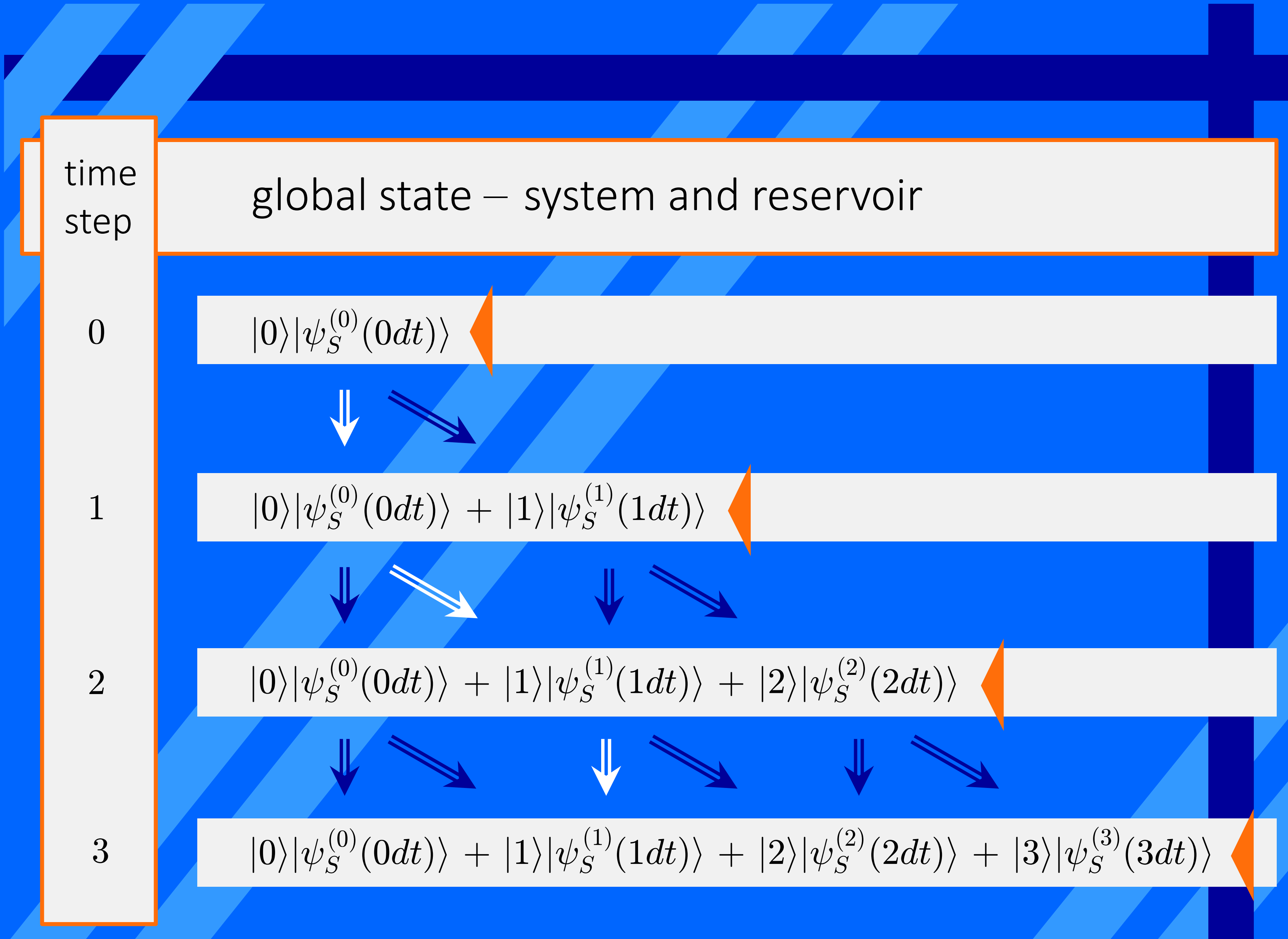
rewritten:

$$\text{Tr}\left\{\underline{\text{exp}[(\mathcal{L} - \mathcal{S})(t - t_n)]}\,\underline{\mathcal{S}} \cdots \underline{\mathcal{S}}\,\underline{\text{exp}[(\mathcal{L} - \mathcal{S})t_1]}\rho(0)\right\} dt_1 \cdots dt_n$$

Master equations, Langevin equations, inputs & outputs

Quantum trajectories – unraveling the master equation

Quantum trajectories – Monte-Carlo simulation


time step
global state – system and reservoir
0
$|0\rangle|\psi_S^{(0)}(0dt)\rangle$
1
$|0\rangle|\psi_S^{(0)}(0dt)\rangle + |1\rangle|\psi_S^{(1)}(1dt)\rangle$
2
$|0\rangle|\psi_S^{(0)}(0dt)\rangle + |1\rangle|\psi_S^{(1)}(1dt)\rangle + |2\rangle|\psi_S^{(2)}(2dt)\rangle$
3
$|0\rangle|\psi_S^{(0)}(0dt)\rangle + |1\rangle|\psi_S^{(1)}(1dt)\rangle + |2\rangle|\psi_S^{(2)}(2dt)\rangle + |3\rangle|\psi_S^{(3)}(3dt)\rangle$

EVOLVE KETS LABELED BY RECORDS

$$|\widetilde{\psi}_{010}\rangle = \underline{\mathrm{exp}\left[-\frac{1}{i\hbar}Hdt\right]J\,\mathrm{exp}\left[-\frac{1}{i\hbar}Hdt\right]|\psi(0)\rangle}$$

EITHER: no-count evolution

$$|\widetilde{\psi}_{0010}\rangle = \mathrm{exp}\left[-\frac{1}{i\hbar}H2dt\right]J\,\mathrm{exp}\left[-\frac{1}{i\hbar}Hdt\right]|\psi(0)\rangle$$

OR: jump

$$|\widetilde{\psi}_{1010}\rangle = J\,\mathrm{exp}\left[-\frac{1}{i\hbar}Hdt\right]J\,\mathrm{exp}\left[-\frac{1}{i\hbar}Hdt\right]|\psi(0)\rangle$$

BRANCHING RATIO – BAYESIAN INFERENCE

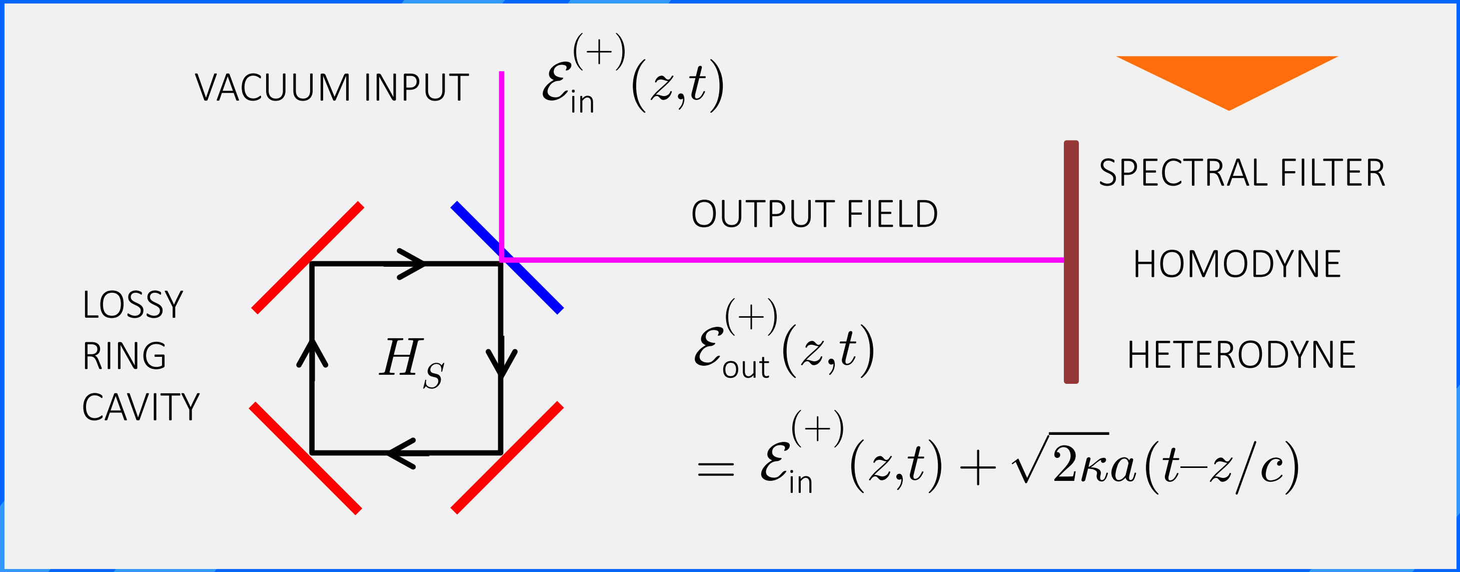
$$\mathrm{Prob}(1\,|\,010) = \frac{\mathrm{Prob}(1 \wedge 010)}{\mathrm{Prob}(010)}$$

$$= \frac{\langle \widetilde{\psi}_{010} |\, J^{\dagger} J \,| \widetilde{\psi}_{010} \rangle dt^2}{\langle \widetilde{\psi}_{010} | \, \widetilde{\psi}_{010} \rangle dt}$$

$$= 2\kappa \langle \psi_{010} | a^{\dagger} a | \psi_{010} \rangle dt$$

MORE UNRAVELINGS

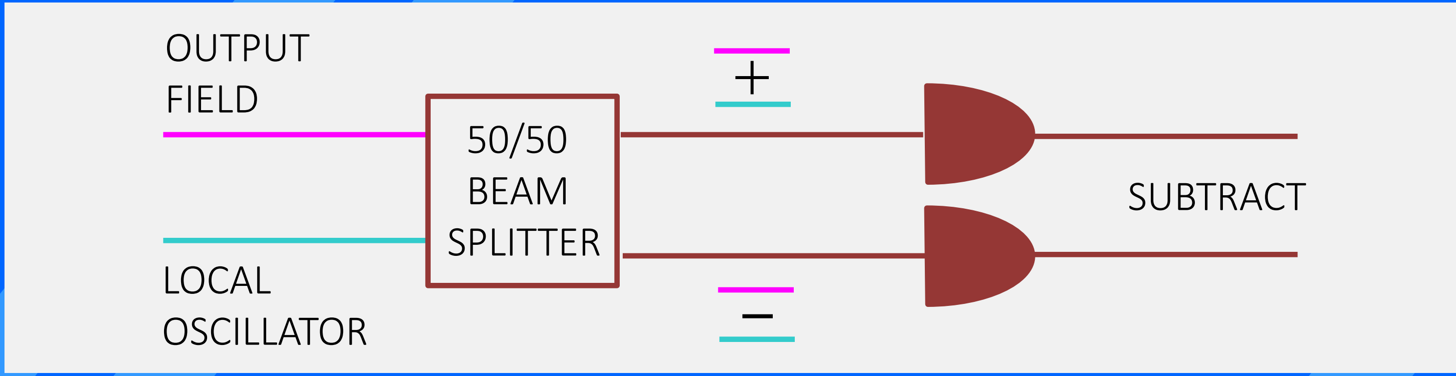
$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H_S, \rho] + \kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)$$



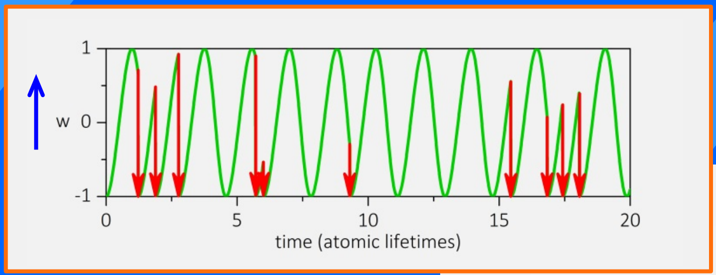
$$d|\tilde{\tilde{\psi}}_{\mathrm{REC}}\rangle = \left[\frac{1}{i\hbar}(H_S - i\hbar\kappa a^\dagger a)dt + \begin{Bmatrix} e^{-i\theta} \\ 1 \end{Bmatrix}\sqrt{2\kappa}\, a\, dq\right]|\tilde{\tilde{\psi}}_{\mathrm{REC}}\rangle$$

with “charge” increment

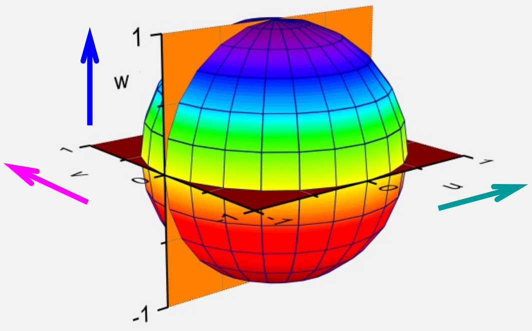
$$dq = \begin{cases} 2\sqrt{2\kappa}\langle X_\theta\rangle_{\mathrm{REC}}dt + dW \\ \sqrt{2\kappa}\langle a^\dagger\rangle_{\mathrm{REC}}dt + dZ \end{cases}$$

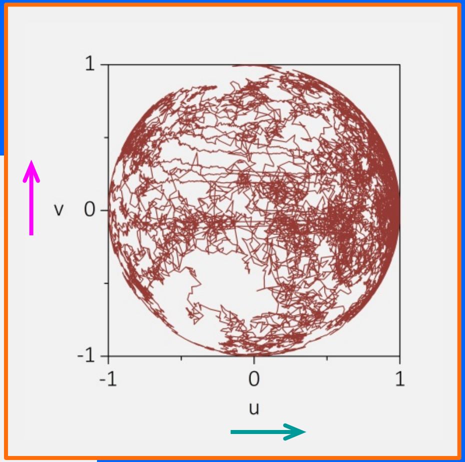


RESONANCE FLUORESCENCE



counting

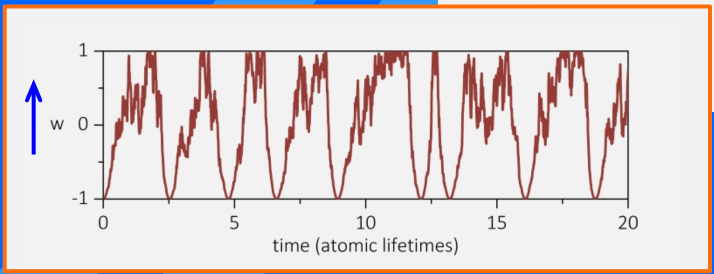


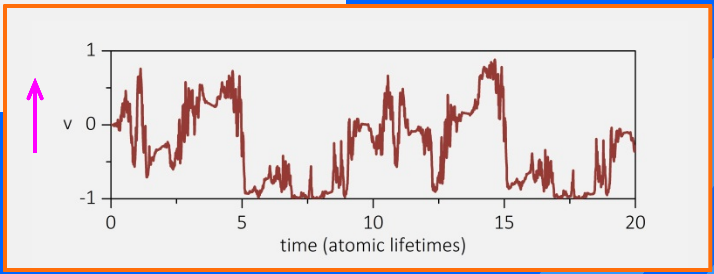


heterodyne

X homodyne

Y homodyne





Open Quantum Systems

Lecture IV:

Master Equations, Inputs & Outputs, Quantum Trajectories – Part B

H. J. Carmichael

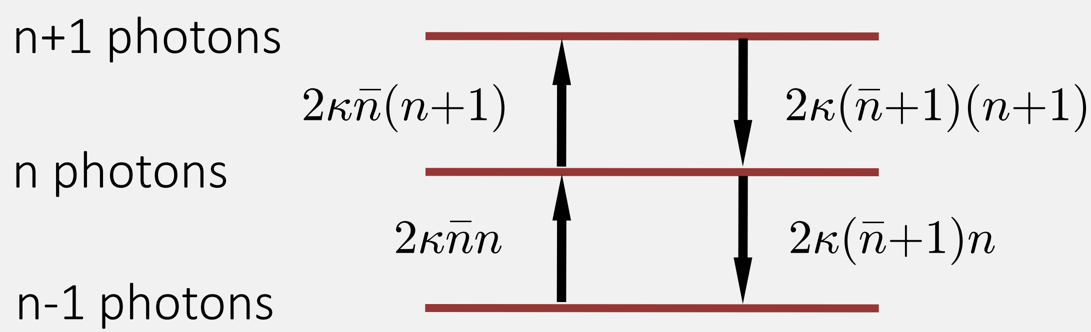
University of Auckland

down transitions

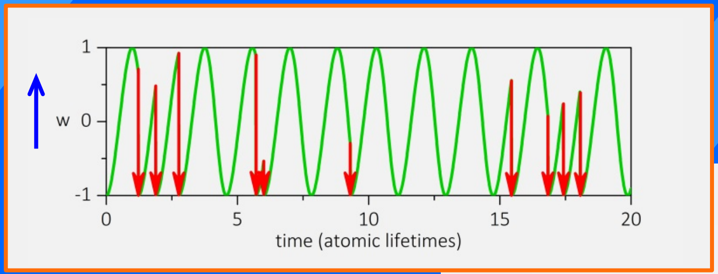
$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H_S, \rho] + \kappa(\bar{n}+1)(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)$$

up transitions

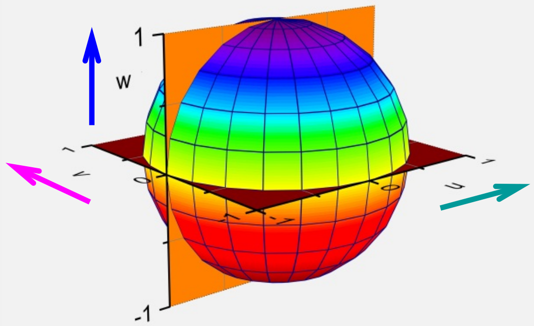
$$+ \kappa\bar{n}(2a^\dagger\rho a - aa^\dagger\rho - \rho aa^\dagger)$$

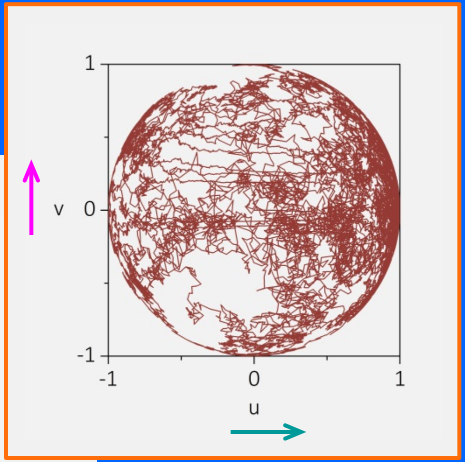


RESONANCE FLUORESCENCE



counting

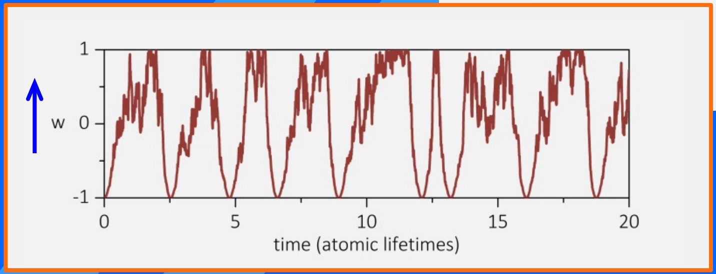


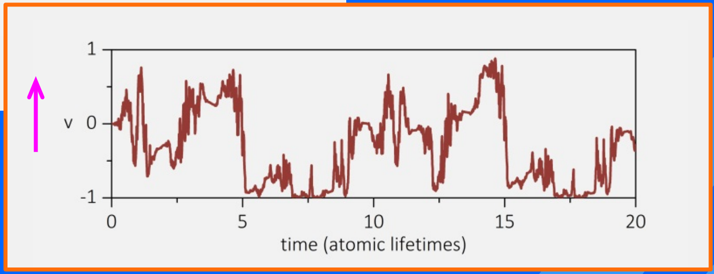


heterodyne

X homodyne

Y homodyne





Jaynes-Cummings versus Einstein A & B

Null-measurement & coherence – electron shelving

Decoherence & measurement – Schrödinger cats

Jaynes-Cummings versus Einstein A & B

Null-measurement & coherence – electron shelving

Decoherence & measurement – Schrödinger cats

Vol 446|15 March 2007|doi:10.1038/nature05589

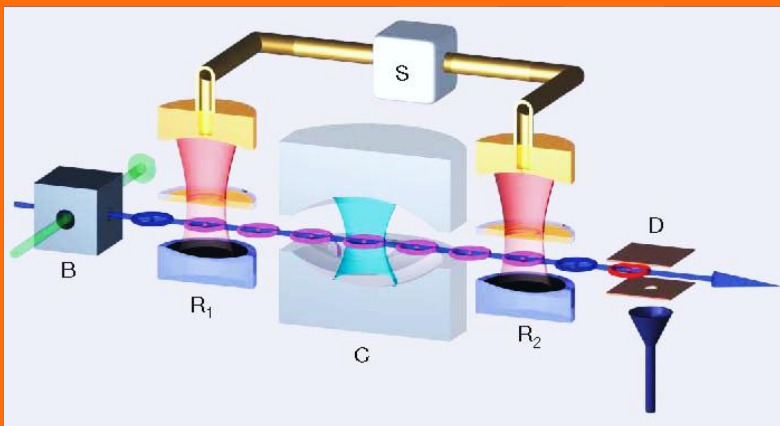
nature

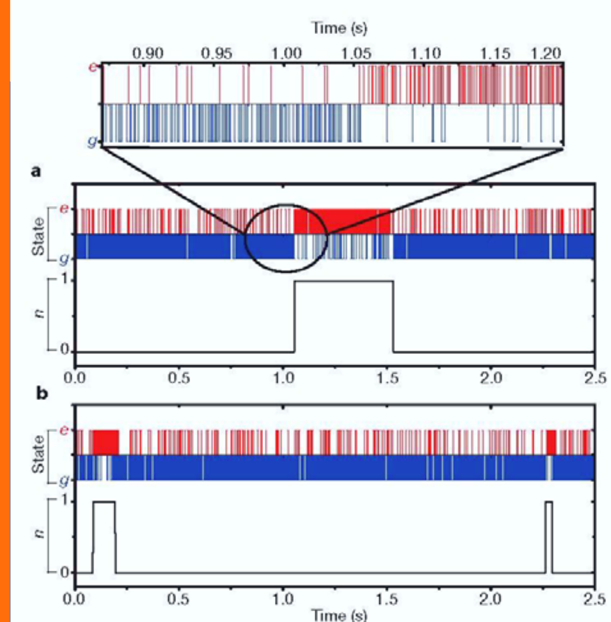
Nature 446, 297 (2007)

LETTERS

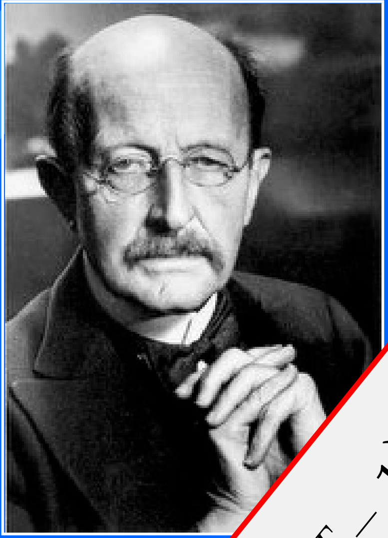
Quantum jumps of light recording the birth and death of a photon in a cavity

Sébastien Gleyzes[1], Stefan Kuhr[1]†, Christine Guerlin[1], Julien Bernu[1],
Michel Brune[1], Jean-Michel Raimond[1] & Serge Haroche[1,2]



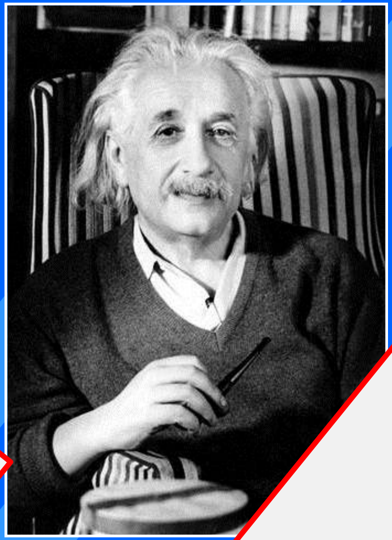


1900



$$U(f) = \frac{8\pi h f^3}{c^3}\left(e^{hf/k_\mathrm{B}T} - 1\right)^{-1}$$

1905



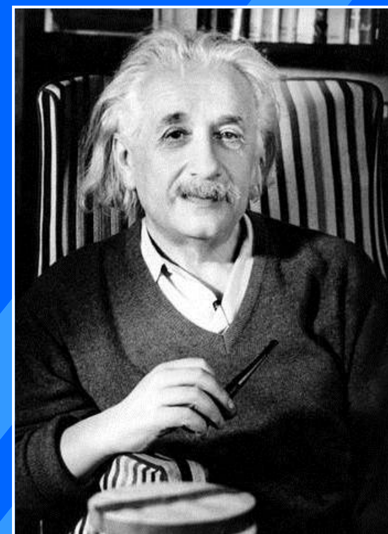
$$eV_\mathrm{s} = hf - \phi$$

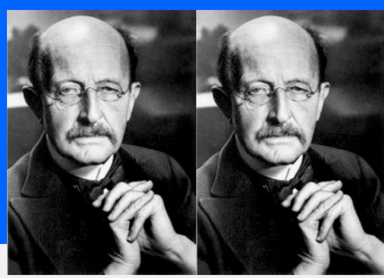
1913

$$hf_\mathrm{eg} = \frac{h^2}{8\pi^2 m a_0^2}\left(n_\mathrm{g}^{-2} - n_\mathrm{e}^{-2}\right)$$

1916

A & B





Planck:
blackbody energy density

$$\frac{8\pi h f_{\mathrm{eg}}^3}{c^3}\left(\underline{e^{h f_{\mathrm{eg}}/k_{\mathrm{B}}T}} - 1\right)^{-1}$$

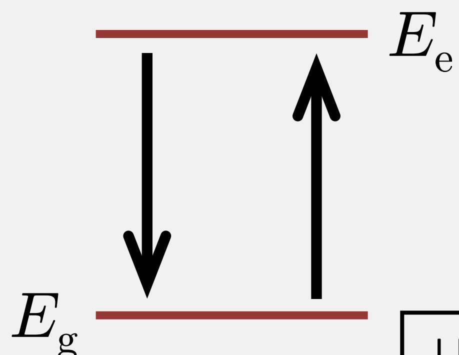
Maxwell-Boltzmann:
equilibrium populations

$$\underline{e^{-(E_{\mathrm{e}}-E_{\mathrm{g}})/k_{\mathrm{B}}T}}$$

SPONTANEOUS EMISSION | STIMULATED EMISSION | ABSORPTION

Down jump rate:

$$BU(f_{\mathrm{eg}})+A = A\bar{n}(f_{\mathrm{eg}})+A$$



Up jump rate:

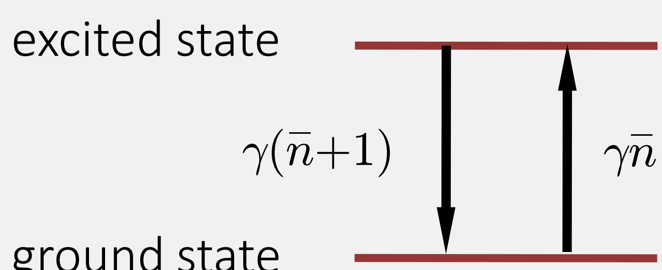
$$BU(f_{\mathrm{eg}}) = A\bar{n}(f_{\mathrm{eg}})$$

down transitions

$$\frac{d\widetilde{\rho}}{dt} = \frac{\gamma}{2}(\bar{n}+1)(2|\mathrm{g}\rangle\langle\mathrm{e}|\widetilde{\rho}|\mathrm{e}\rangle\langle\mathrm{g}| - |\mathrm{e}\rangle\langle\mathrm{e}|\widetilde{\rho} - \widetilde{\rho}|\mathrm{e}\rangle\langle\mathrm{e}|)$$

up transitions

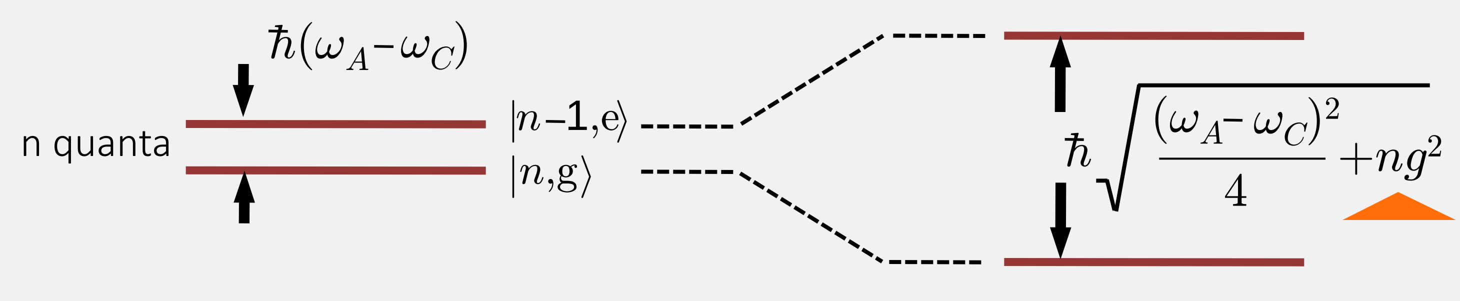
$$+\frac{\gamma}{2}\bar{n}(2|\mathrm{e}\rangle\langle\mathrm{g}|\widetilde{\rho}|\mathrm{g}\rangle\langle\mathrm{e}| - |\mathrm{g}\rangle\langle\mathrm{g}|\widetilde{\rho} - \widetilde{\rho}|\mathrm{g}\rangle\langle\mathrm{g}|)$$



JAYNES-CUMMINGS MODEL

$$H_{JC} = \underbrace{\hbar\omega_C a^\dagger a + \frac{\hbar\omega_A}{2}(|\mathrm{e}\rangle\langle\mathrm{e}| - |\mathrm{g}\rangle\langle\mathrm{g}|)}_{} \quad \text{free cavity + atom}$$

$$+ \underbrace{\hbar g(a^\dagger|\mathrm{g}\rangle\langle\mathrm{e}| + a|\mathrm{e}\rangle\langle\mathrm{g}|)}_{} \quad \text{dipole coupling}$$



JUMPS FROM A COHERENT INTERACTION?

$$H = \underline{E_{\mathrm{e}}|E_{\mathrm{e}}\rangle\langle E_{\mathrm{e}}| + E_{\mathrm{g}}|E_{\mathrm{g}}\rangle\langle E_{\mathrm{g}}|}$$

$$\underline{+\hbar\sum_j \omega_j r_j^\dagger r_j + \hbar\sum_j \left(\kappa_j r_j|E_{\mathrm{e}}\rangle\langle E_{\mathrm{g}}| + \mathrm{h.c.}\right)}$$

All modes but one make atom jump…

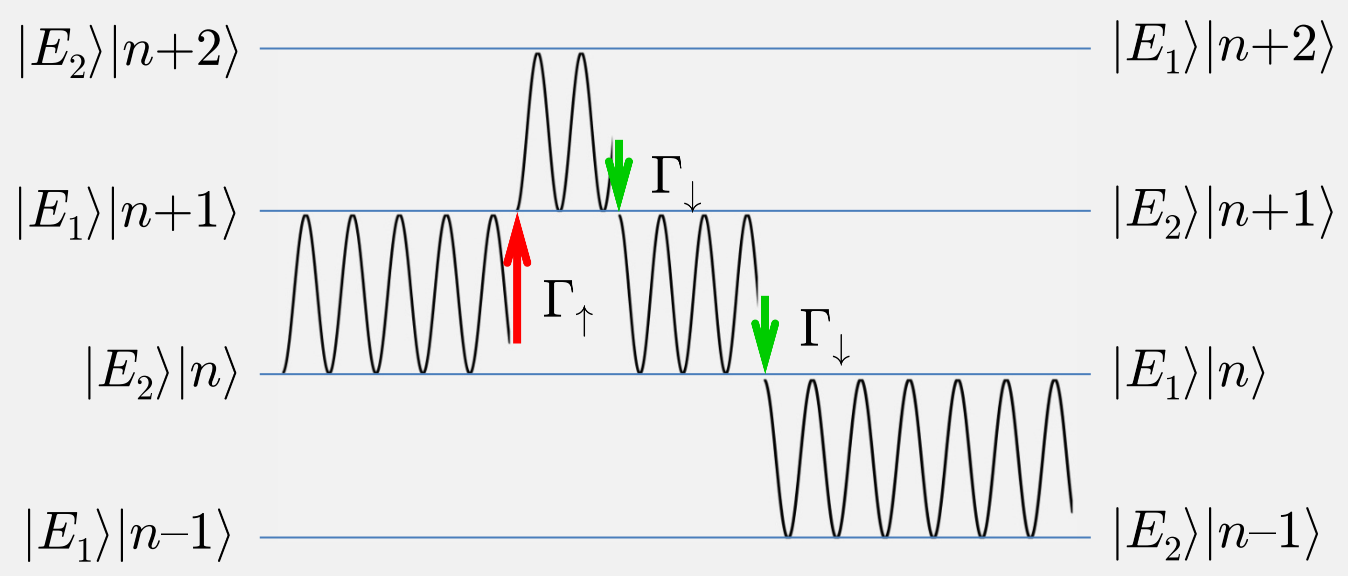
$$H = (E_{\mathrm{e}} - \underline{i\Gamma_\downarrow/2})|E_{\mathrm{e}}\rangle\langle E_{\mathrm{e}}| + (E_{\mathrm{g}} - \underline{i\Gamma_\uparrow/2})|E_{\mathrm{g}}\rangle\langle E_{\mathrm{g}}|$$

$$+\underline{\hbar\omega r^\dagger r + \hbar\left(\kappa r|E_{\mathrm{e}}\rangle\langle E_{\mathrm{g}}| + \mathrm{h.c.}\right)}$$

…does the one mode jump?

STRONG COUPLING – $\lambda/\Gamma_{\uparrow,\downarrow} \gg 1$

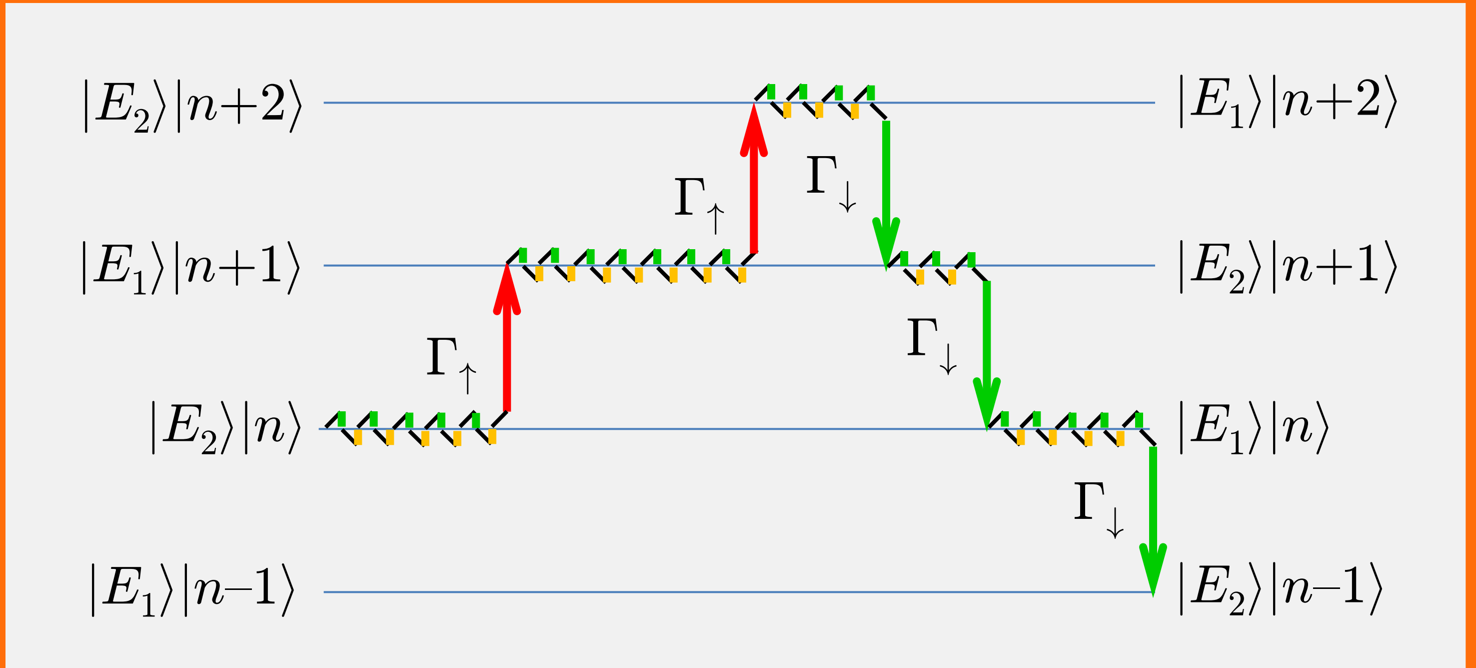
$$\alpha(t)|E_2\rangle|n\rangle + \beta(t)|E_1\rangle|n+1\rangle$$



WEAK COUPLING – $\lambda/\Gamma_{\uparrow,\downarrow} \ll 1$

$|E_2\rangle|n\rangle + \epsilon(t)|E_1\rangle|n{+}1\rangle$

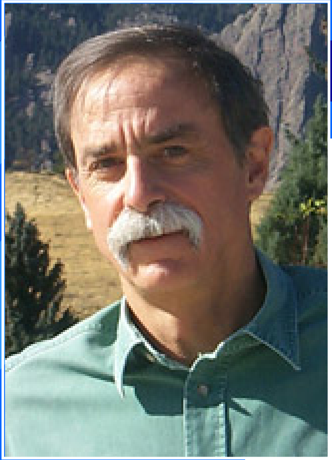
$|E_1\rangle|n\rangle + \epsilon(t)|E_2\rangle|n{-}1\rangle$



Jaynes-Cummings versus Einstein A & B

Null-measurement & coherence – electron shelving

Decoherence & measurement – Schrödinger cats



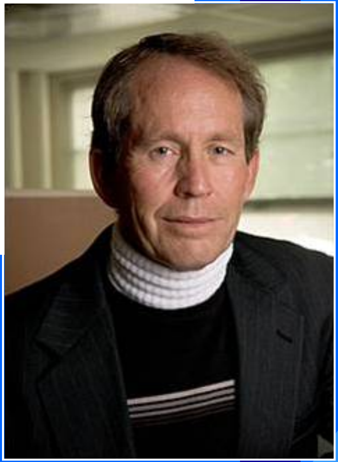
VOLUME 57, NUMBER 14 PHYSICAL REVIEW LETTERS 6 OCTOBER 1986

Observation of Quantum Jumps in a Single Atom

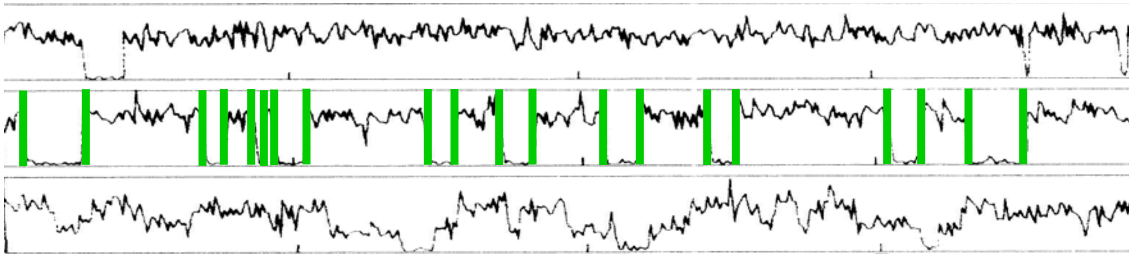
J. C. Bergquist, Randall G. Hulet, Wayne M. Itano, and D. J. Wineland
Time and Frequency Division, National Bureau of Standards, Boulder, Colorado 80303
(Received 23 June 1986)

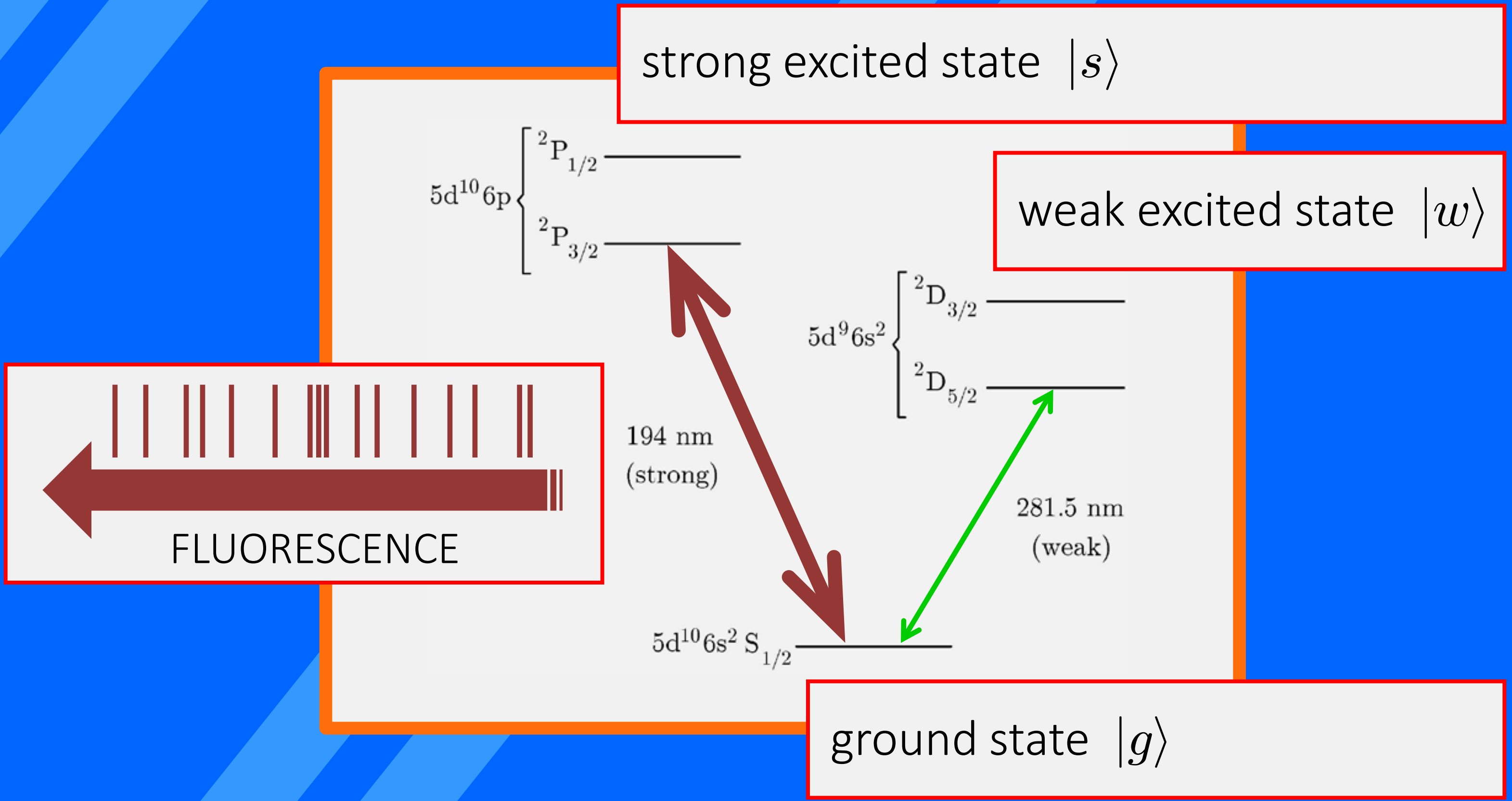
We detect the radiatively driven electric quadrupole transition to the metastable $^2D_{5/2}$ state in a single, laser-cooled Hg II ion by monitoring the abrupt cessation of the fluorescence signal from the laser-excited $^2S_{1/2} \rightarrow {}^2P_{1/2}$ first resonance line. When the ion "jumps" back from the metastable D state to the ground S state, the $S \rightarrow P$ resonance fluorescence signal immediately returns. The statistical properties of the quantum jumps are investigated; for example, photon antibunching in the emission from the D state is observed with 100% efficiency.

PACS numbers: 32.80.Pj, 42.50.Dv



Intermittent fluorescence
of a laser driven mercury ion:

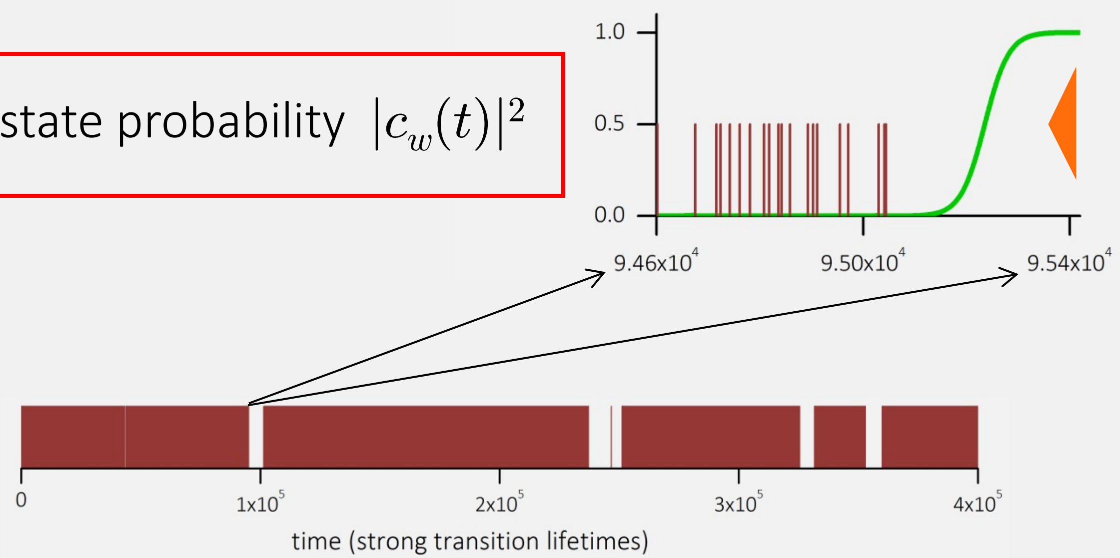




$$c_g(t)|g\rangle + c_s(t)|s\rangle + c_w(t)|w\rangle$$

QUANTUM TRAJECTORY SIMULATION

shelved state probability $|c_w(t)|^2$



$$c_g(t)|g\rangle + c_s(t)|s\rangle + c_w(t)|w\rangle$$

NULL MEASUREMENT & BAYESIAN INFERENCE

$$|\psi(0)\rangle = c_{\mathrm{g}}|\mathrm{g}\rangle + c_{\mathrm{e}}|\mathrm{e}\rangle$$

$$|\tilde{\psi}_{0\ldots0}(t)\rangle = \exp\left[-\left(\frac{\gamma}{2} + i\omega_A\right)(|\mathrm{e}\rangle\langle\mathrm{e}|)t\right]|\psi(0)\rangle$$

$$= c_{\mathrm{g}}|\mathrm{g}\rangle + c_{\mathrm{e}}e^{-(\gamma/2+i\omega_A)t}|\mathrm{e}\rangle$$

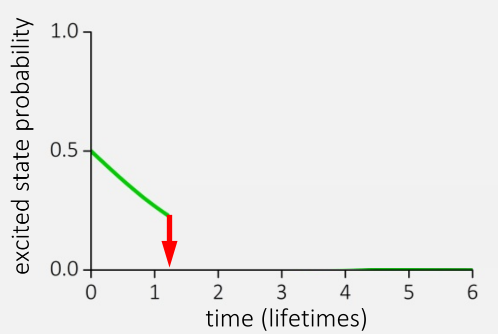
$$\langle\tilde{\psi}_{0\ldots0}|\tilde{\psi}_{0\ldots0}\rangle = |c_{\mathrm{g}}|^2 + e^{-\gamma t}|c_{\mathrm{e}}|^2$$

$$|\psi_{0\ldots0}(t)\rangle = \frac{c_{\mathrm{g}}|\mathrm{g}\rangle + c_{\mathrm{e}}e^{-(\gamma/2+i\omega_A)t}|\mathrm{e}\rangle}{\sqrt{|c_{\mathrm{g}}|^2 + e^{-\gamma t}|c_{\mathrm{e}}|^2}}$$

$$\mathrm{Prob}(1\,|\,0\ldots0) = \gamma dt\,\frac{e^{-\gamma t}|c_{\mathrm{e}}|^2}{|c_{\mathrm{g}}|^2 + e^{-\gamma t}|c_{\mathrm{e}}|^2}$$

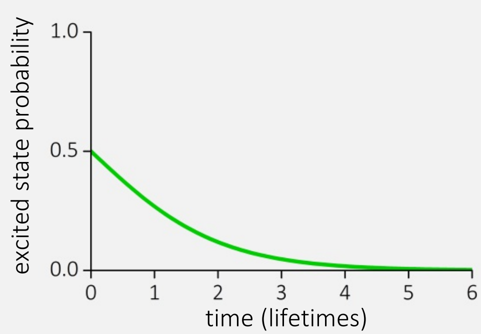
EITHER:

50%



OR:

50%



Jaynes-Cummings versus Einstein A & B

Null-measurement & coherence – electron shelving

Decoherence & measurement – Schrödinger cats

VOLUME 77, NUMBER 24 PHYSICAL REVIEW LETTERS 9 DECEMBER 1996

Observing the Progressive Decoherence of the "Meter" in a Quantum Measurement

M. Brune, E. Hagley, J. Dreyer, X. Maître, A. Maali, C. Wunderlich, J. M. Raimond, and S. Haroche
Laboratoire Kastler Brossel, Département de Physique de l'Ecole Normale Supérieure, 24 Rue Lhomond, F-75231 Paris Cedex 05, France*
(Received 10 September 1996)

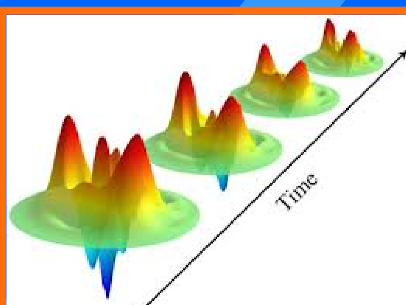
nature Vol 455|25 September 2008|doi:10.1038/nature07288

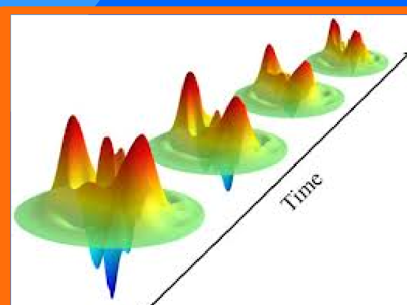
LETTERS

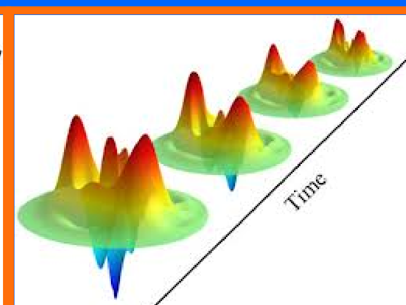
Nature 455, 510 (2008)

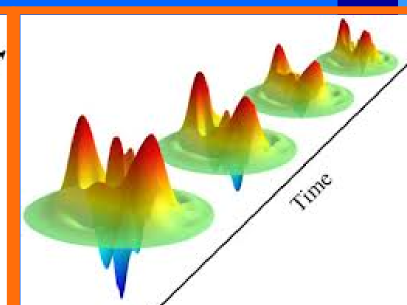
Reconstruction of non-classical cavity field states with snapshots of their decoherence

Samuel Deléglise[1], Igor Dotsenko[1,2], Clément Sayrin[1], Julien Bernu[1], Michel Brune[1], Jean-Michel Raimond[1] & Serge Haroche[1,2]



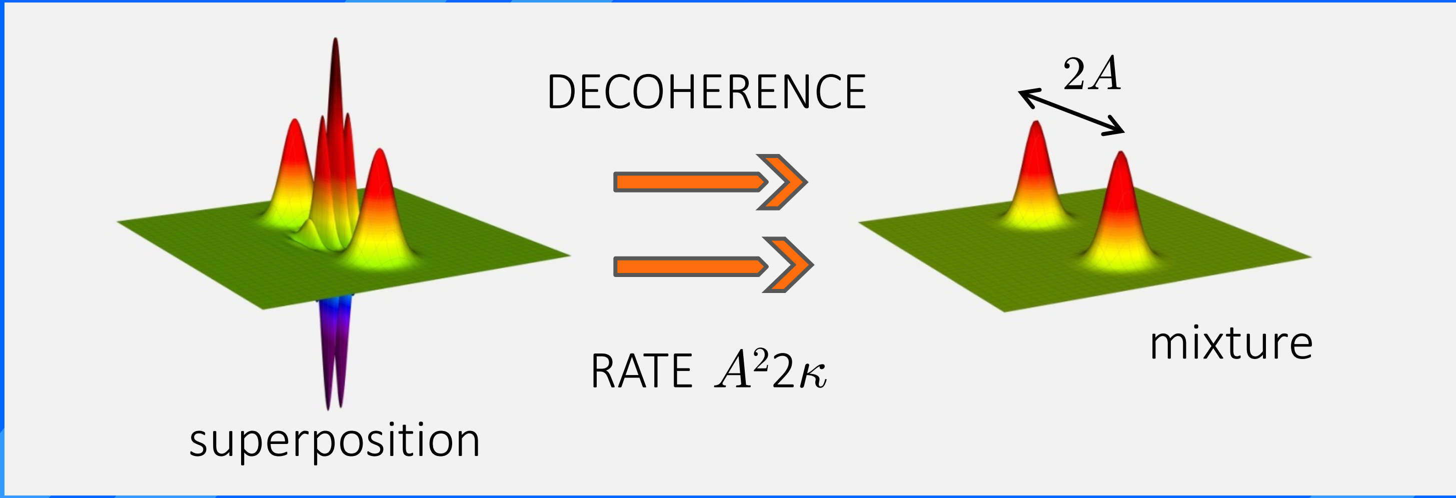






DECAY OF COHERENT STATE SUPERPOSITION

$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H_S, \rho] + \underline{\kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)}$$

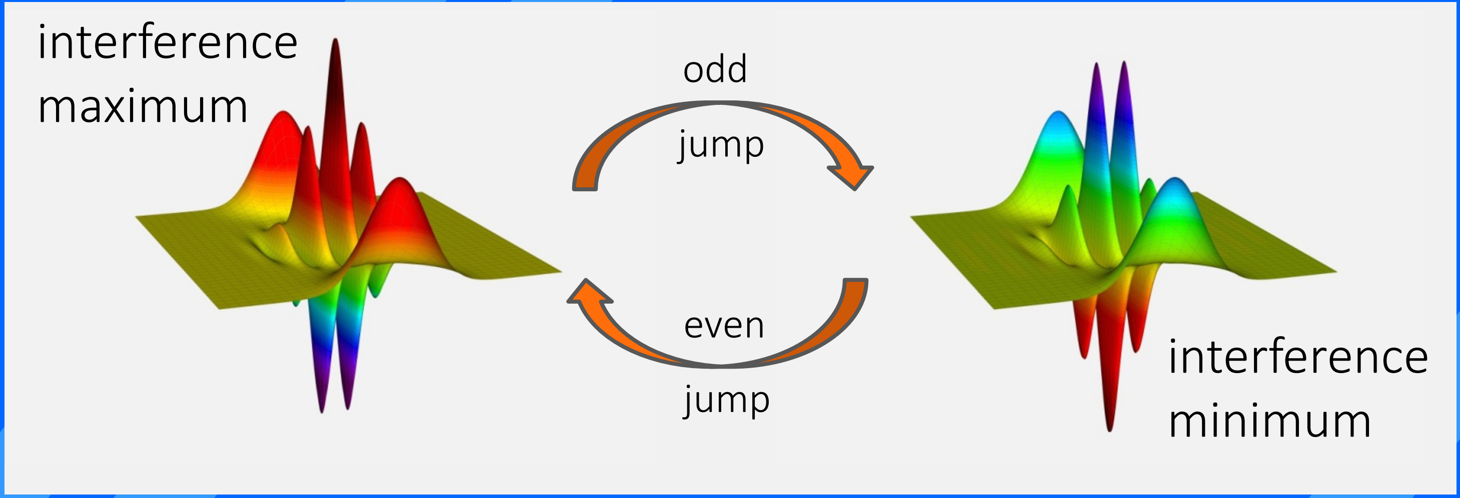


COUNTING QUANTUM TRAJECTORY

!! first lost photon kills the cat !!

(inside the cavity)

$$|Ae^{-\kappa t}\rangle + |-Ae^{-\kappa t}\rangle \qquad\qquad |Ae^{-\kappa t}\rangle - |-Ae^{-\kappa t}\rangle$$

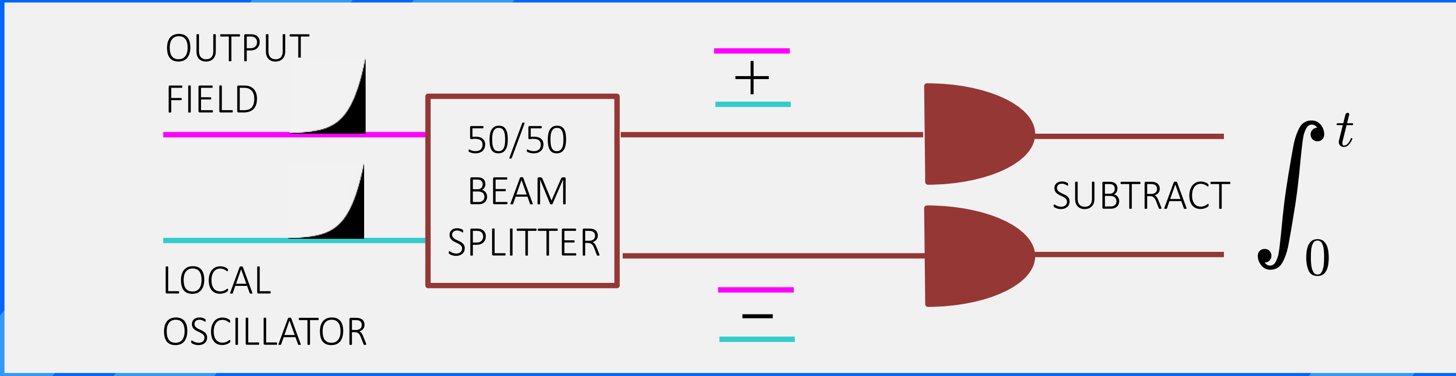


HOMODYNE QUANTUM TRAJECTORY

$$d|\tilde{\tilde{\psi}}_{\mathrm{REC}}\rangle = \Big[-\kappa a^{\dagger}a\,dt + e^{-i\theta}\sqrt{2\kappa}\,a\,dq_t\Big]|\tilde{\tilde{\psi}}_{\mathrm{REC}}\rangle$$

with "charge" increment

$$dq_t = 2\sqrt{2\kappa}\langle X_\theta\rangle_{\mathrm{REC}}dt + dW$$



!! last lost photon revives the cat !!

(outside the cavity)

$$e^{+z(t)}|+Ae^{-\kappa t}\rangle + e^{-z(t)}|-Ae^{-\kappa t}\rangle \qquad z(t) = \phi(t)e^{i\theta}$$

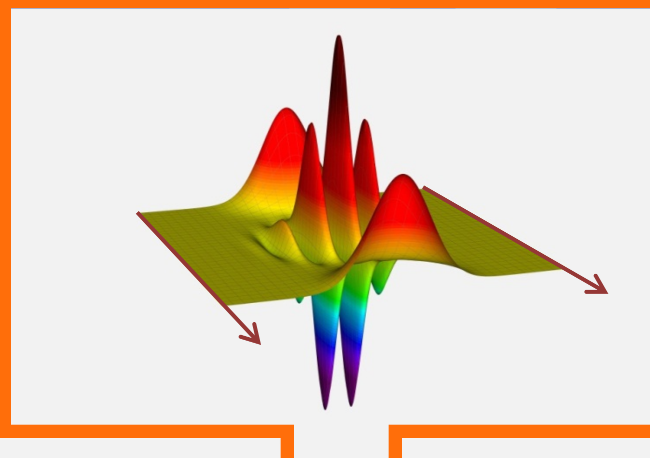
Measurement record:

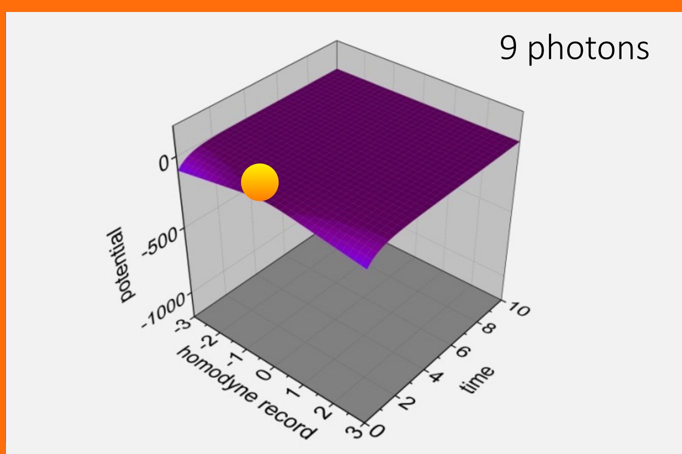
$$\phi(t) = \int_0^t e^{-\kappa t'} dq_{t'}$$

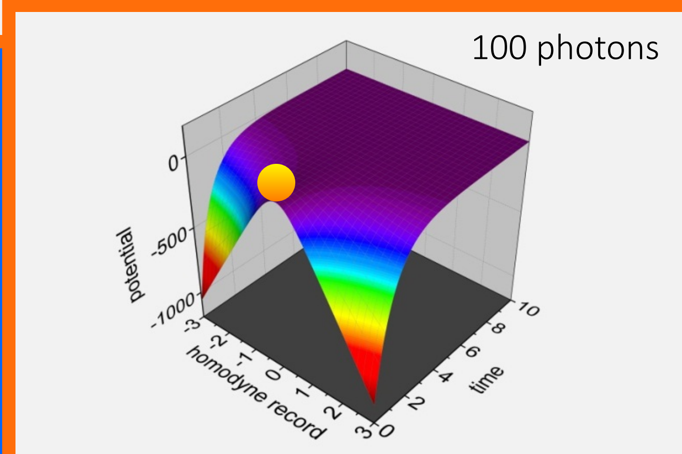
with

$$d\phi = -\frac{\partial}{\partial\phi}\underbrace{V(\phi,t)}_{\text{time-dependent potential}}dt + \underbrace{dW}_{\text{shot noise}}$$

LOCAL OSCILLATOR PHASE $= 0$







LOCAL OSCILLATOR PHASE $= \pi/2$